A Particle Flow Filter for Estimating Future Earthquake Occurrences

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Introduction

*Our ability to forecast earthquakes is hampered by limited information of the state of stress, strength, and velocities of faults which is largely unknown and inaccessible. **Data assimilation offers as a means to estimate these inaccessible variables by combining physics-based models and observations taking into account their uncertainties.** This is particularly useful for the field of earthquake forecasting when considering earthquakes as deterministic chaotic process as it proofs to be the case for slow slip events (SSEs) as in **Fig. 1**.*

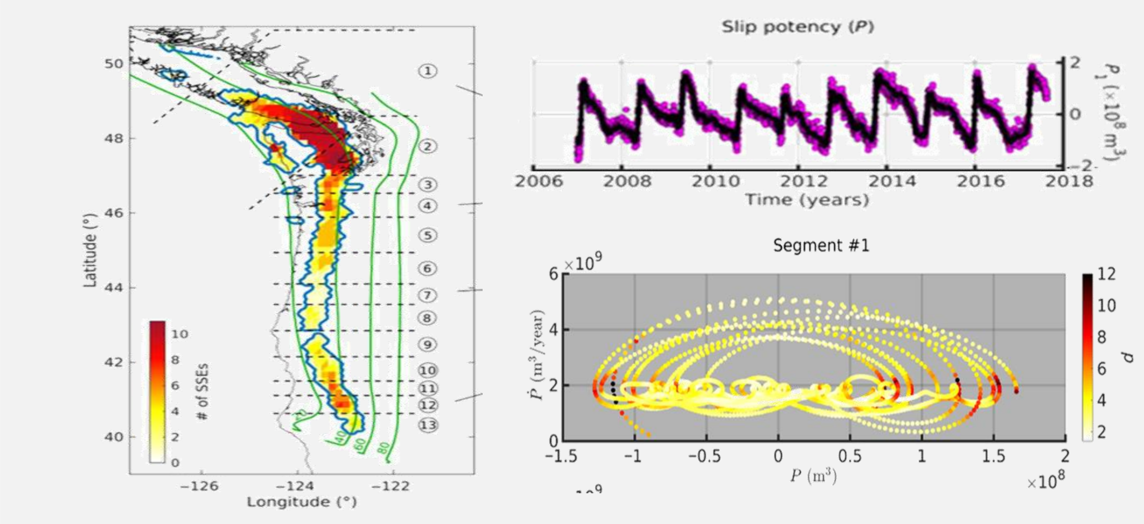


Figure 1. A section of an attractor found by post-processing of slow slip events in a segment of Cascadia . Adapted from Gualandi et al. 2020

*In Diab-Montero et al. 2023, we used an Ensemble Kalman Filter (EnKF) to estimate the shear stress, velocities and state θ in perfect model experiments of a 1D model of a horizontal straight fault that generated earthquakes and SSEs. The results were satisfactory but we noticed the presence of non-Gaussian priors closely before and after the coseismic phase of the earthquakes (**Fig .2**) that may challenge the accuracy of the EnKF. **In this study we compare the estimates of the EnKF with a Particle Flow filter (PFF) to assess whether these priors impact or not the accuracy of the estimates.***

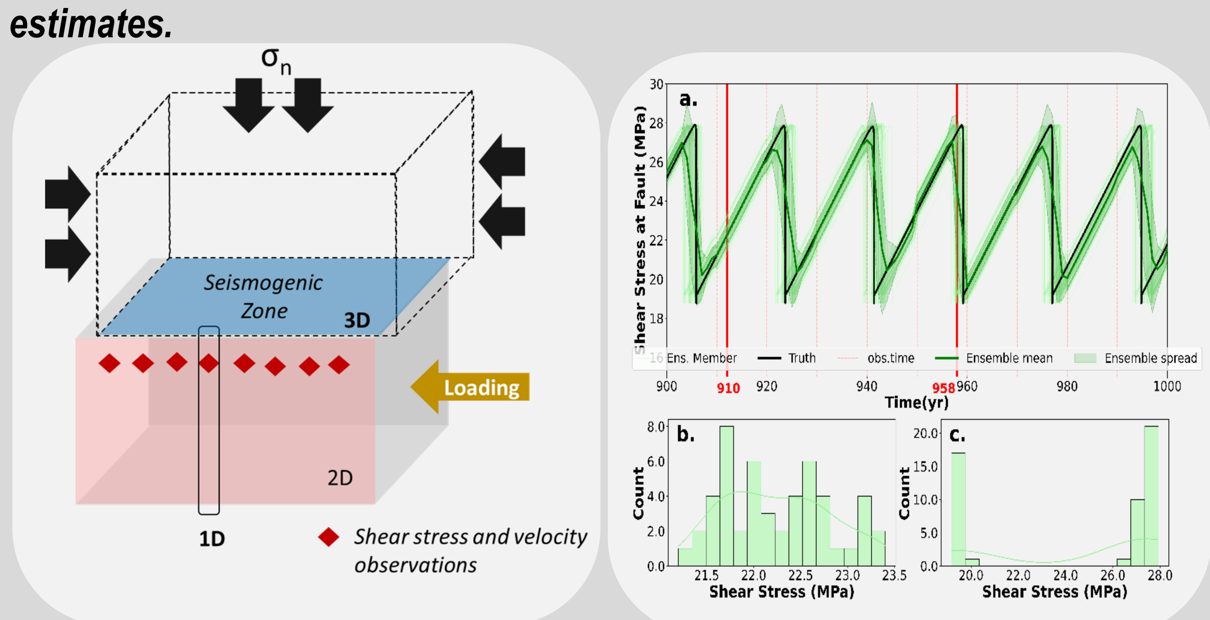


Figure 2. Schematic diagram of horizontal straight fault model and ensemble distribution of shear stress for the interseismic and coseismic phase

Data Assimilation Methods

The EnKF updates the ensemble from its prior to the posterior in a single analysis step as follows:

$$\boldsymbol{z}_n^a = \boldsymbol{z}_n^f + \left(\boldsymbol{C}_{zz}^f \boldsymbol{H}^T\right)\left(\boldsymbol{H}\boldsymbol{C}_{zz}^f \boldsymbol{H}^T + \boldsymbol{C}_{yy}\right)^{-1}\left(\boldsymbol{y}_n - \boldsymbol{H}\boldsymbol{z}_n^f\right), \qquad 1 \leq n \leq N$$

*The PFF is a non-Gaussian data assimilation method that estimates sequential updates that make the ensemble members to "flow" in a pseudo-time **s** from the prior to the posterior distribution following these equations:*

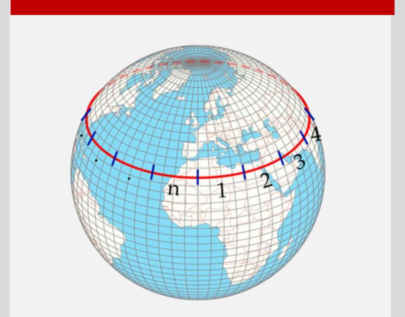
$$\frac{d}{ds} z_s = f_s(z_s), \qquad s \in [0, \infty]$$

$$f_s(z) = \frac{1}{N} D \sum_{n=1}^{N} \left\{ \mathbf{K}(\mathbf{z}_s^n, \mathbf{z}) \nabla log p(\mathbf{z}_s^n | y) + \nabla_{\mathbf{z}_s^n} \cdot \mathbf{K}(\mathbf{z}_s^n, \mathbf{z}) \right\}$$

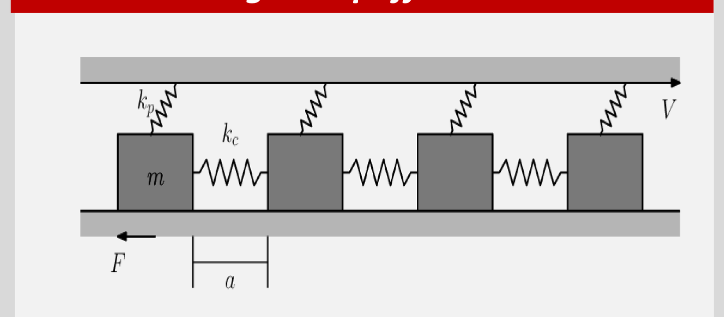
Forward Models

In our study we compare the performance of the EnKF and the PFF on the Lorenz 96 model of 20 variables, a benchmark model commonly used in atmospheric sciences, with a Burridge Knopoff 1D model of 20 blocks coupled with rate-and-state friction, a model used by seismologists to understand seismic cycles. To be more precise we use the non-dimensional set of equations of Erickson et al. 2011.

Lorenz 96



Burridge-knopoff 1D model



$$\frac{dx_i}{dt} = (x_{i+1} - x_{i-2})x_{i-1} - x_i + F$$

$$x_0 = x_N$$
$$x_{N+1} = x_1$$
$$x_{-1} = x_{N-1}$$

$$\dot{\bar{u}}_j = \bar{v}_j$$

$$\dot{\bar{v}}_j = \gamma_\mu^2(\bar{u}_{j-1} - 2\bar{u}_j + \bar{u}_{j+1}) - \gamma_\lambda^2 \bar{u}_j - \left(\gamma_\mu^2/\xi\right)\left(\bar{f} + \bar{\Theta}_j + \ln(\bar{v}_j + 1)\right)$$

$$\dot{\bar{\Theta}}_j = -(\bar{v}_j + 1)(\bar{\Theta}_j + (1 + \epsilon)\ln(\bar{v}_j + 1))$$

Perfect Model Experiments

*We perform perfect model experiments on the Lorenz 96 and the Burridge Knopoff 1D model under periodic and chaotic conditions as in **Fig. 3** to understand how periodicity and aperiodicty affects the performance of the filters in the state estimation task.*

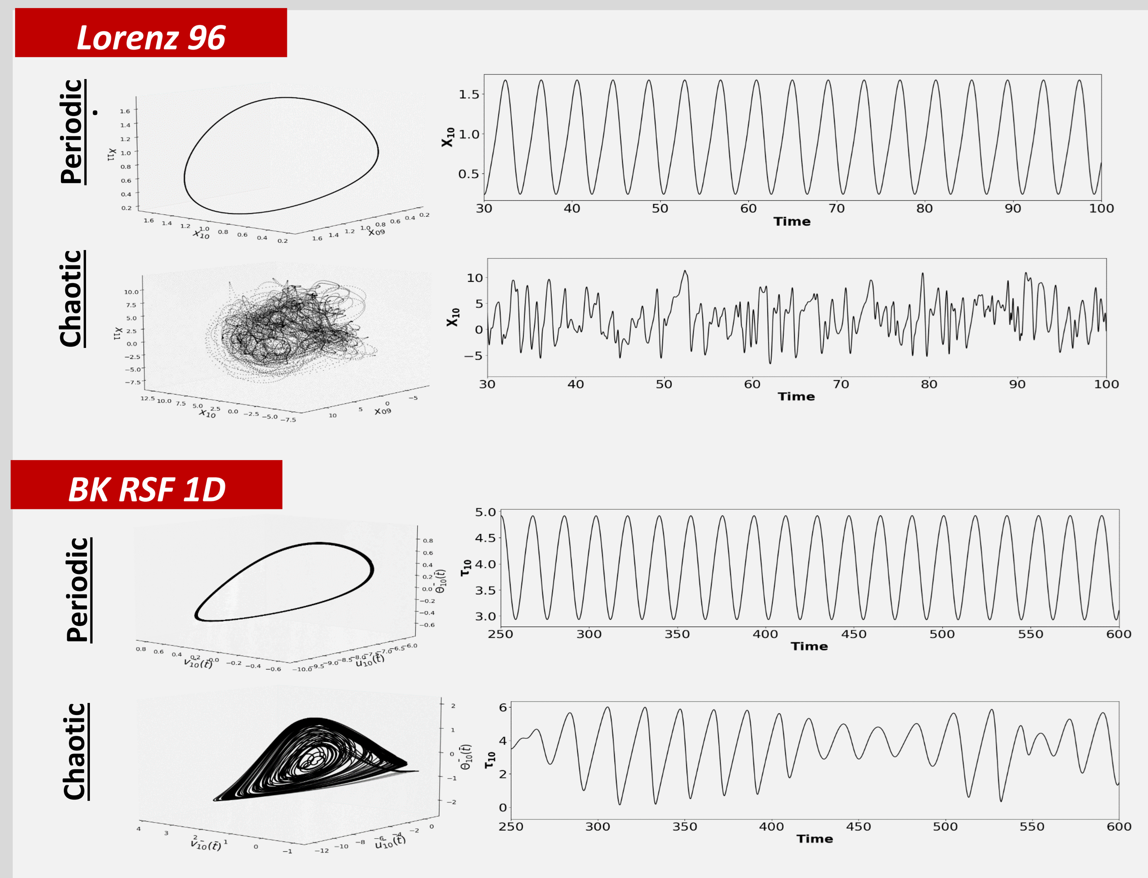


Figure 3. Phase diagrams and examples of time series for the Lorenz 96 of 20 variables and the Burridge Knopoff 1D model under periodic and chaotic conditions

*In the perfect model experiments we consider three different scenarios (1) Observing a different percentage of the total grid points (Obs. density) as in **Fig. 4.** (2) Different observation rates in time, and (3) Different observation errors for the observed variables.*

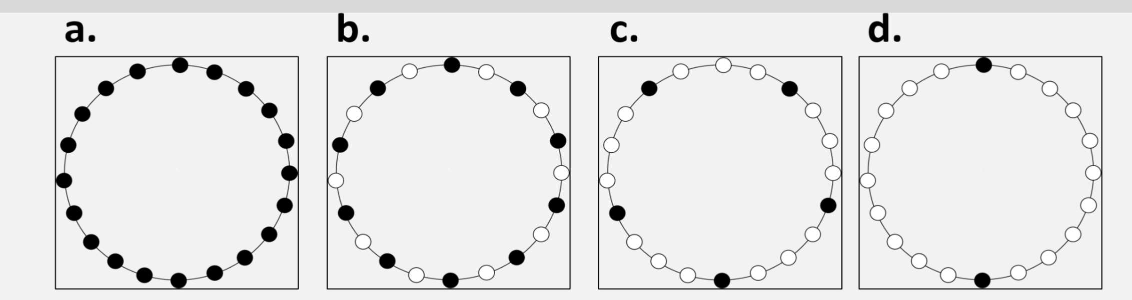


Figure 4. Schematic representation of different observations densities of perfect model experiments for scenario of 1: (a) 100%, (b) 50%, (c) 25% and (d) 10% coverage.

Results 1D Quasi-dynamic model

*We first estimate with the PFF the posterior for the non-Gaussian priors shown in **Fig. 2.**to compare with the EnKF and see if we can obtain better estimates. It is important to mention that we only observe shear stress and velocity at a single location away of the fault.*

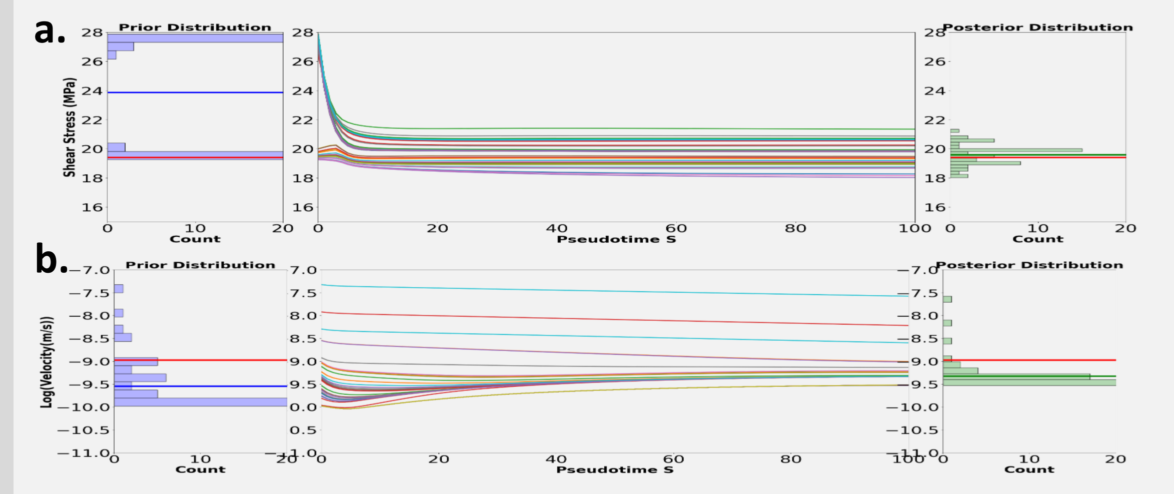


Figure 5. Pseudoflow of particles from the prior to the posterior distribution for a single data assimilation step for the shear (a) stress and (b) velocity.

*We also estimate the posterior for all the other assimilation steps of Diab-Montero et al. 2023 and compare the error of the posterior for the PFF and the ENKF with respect to the truth (**Fig. 6**).*

Figure 6. Comparison of the absolute errors of the EnKF and PFF estimates with respect to the truth for (a) shear stress, (b) velocity and (c) state θ.

The results from both the EnKF and the PFF are very close to each other showing that even when having non-Gaussian priors in this system the estimates of the posterior from the EnKF are close to the truth.

Results Lorenz 96

The results of the perfect model tests with different percentages of observed variables for the Lorenz 96 model show that the PFF provides better estimates than the EnKF in accordance to Hu & van Leeuwen 2021.

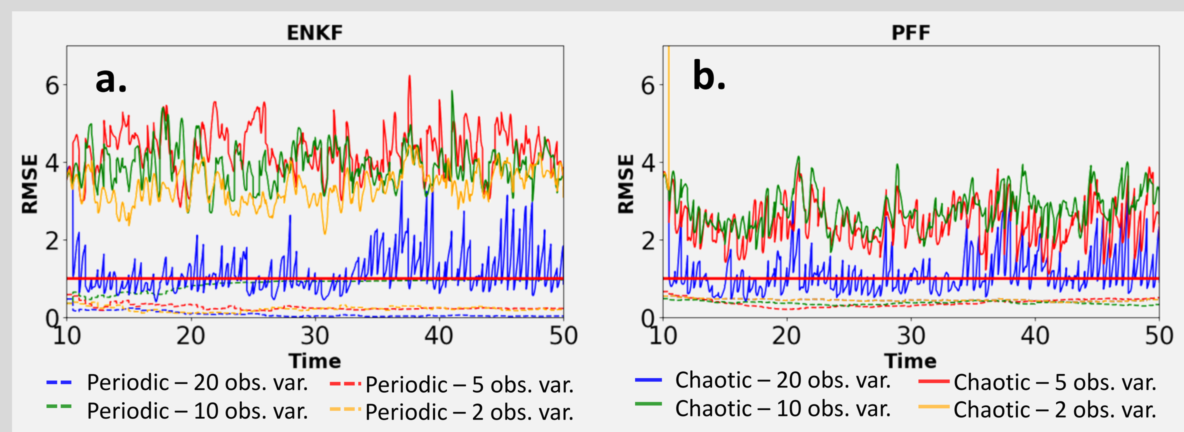


Figure 7. Comparison of the RMSE estimates of the (a) ENKF and the (b) PFF for the Lorenz 96 with 20 variables for different observation coverage.

Results Burridge Knopoff RSF 1D

*We now compare the results for the PFF and the EnKF for the BK model coupled with rate-and-state friction in periodic and chaotic conditions (**Figs. 8 and 9**). The PFF tends to give better estimates of θ which is an unobserved variable. However, the EnKF provides in general better estimates for the shear stress and velocity variables which are observed.*

Periodic

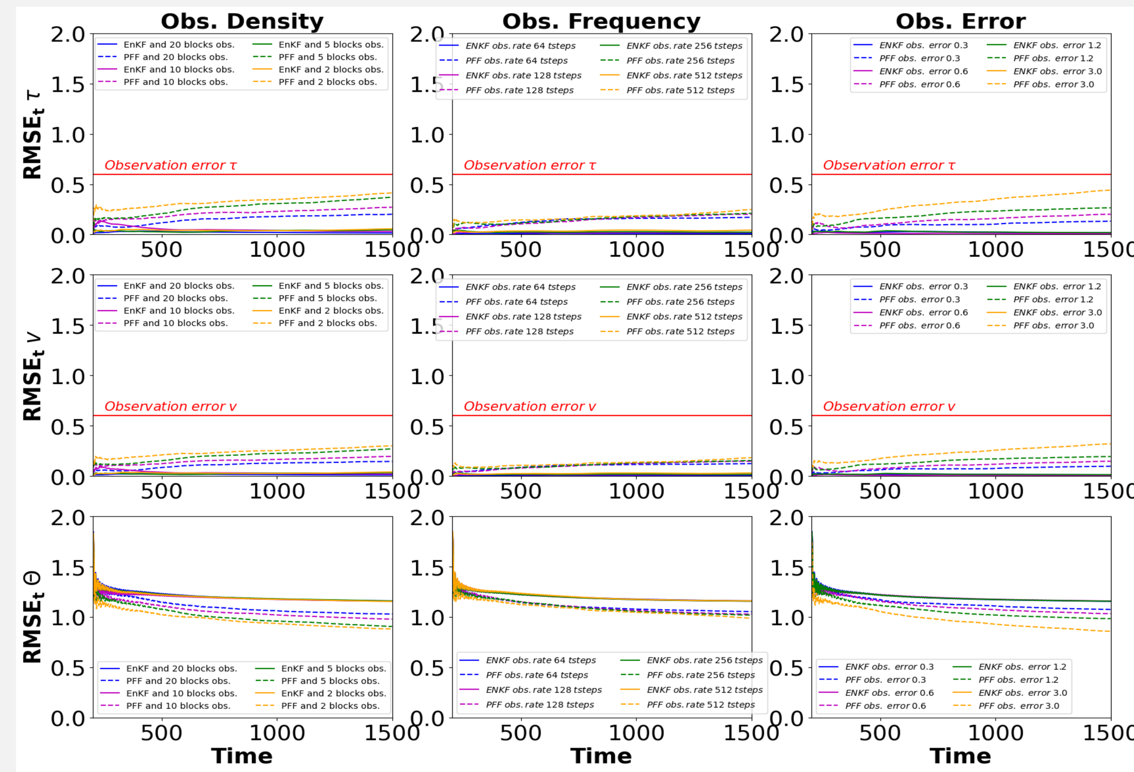


Figure 8. Comparison of the total RMSE of the PFF (dashed lines) and the ENKF (solid lines) for the three scenarios for the periodic Burridge Knopoff 1D model.

Chaotic

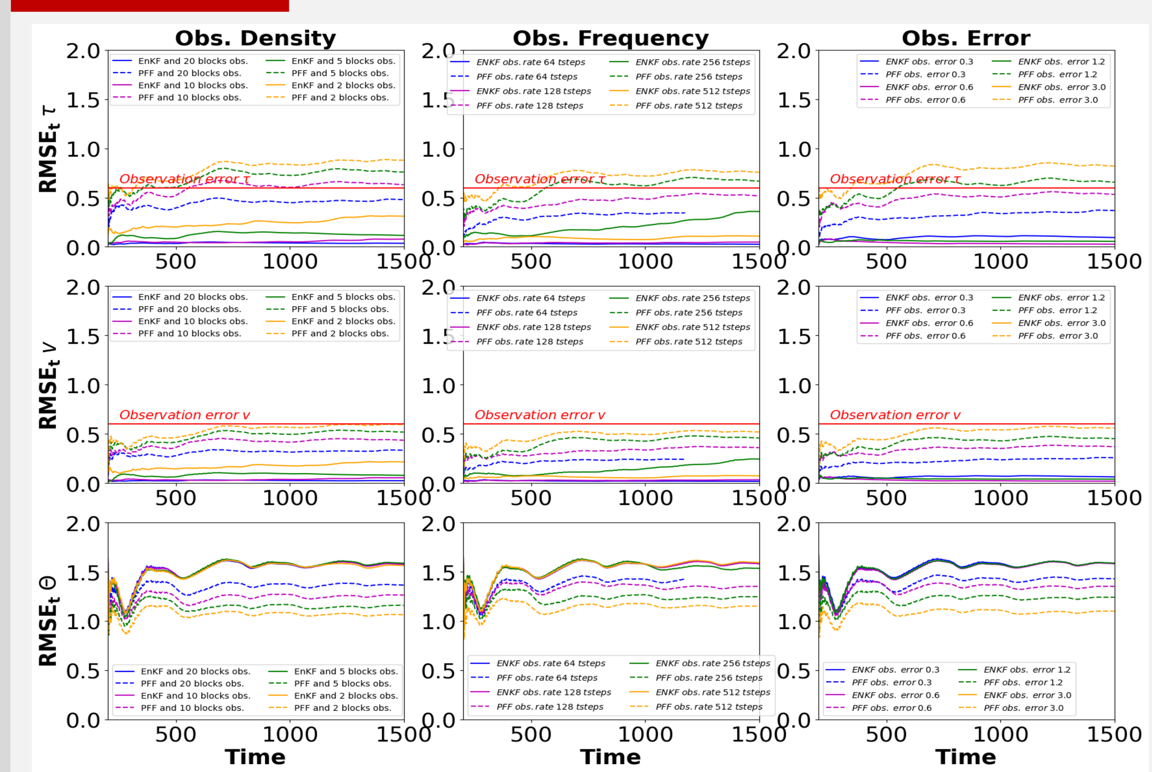


Figure 9. Comparison of the total RMSE of the PFF (dashed lines) and the ENKF (solid lines) for the three scenarios for the chaotic Burridge Knopoff 1D model

Conclusions & Future Work

We continue the work of Diab-Montero et al. 2023 by benchmarking and comparing the estimates of the EnKF with a non-Gaussian and non-linear data assimilation method (PFF).

The results show that the EnKF provides good estimates for a system governed by rate-and-state friction even in chaotic and aperiodic conditions. The results also suggest that the PFF tends to provide more accurate estimates of unobserved variables than the EnKF at the additional cost of the sequential iterations in pseudotime.

We look forward to further test the EnKF and PFF in 2D/3D earthquake models and assimilating laboratory measurements of strain and velocity.

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