Hybrid covariance super-resolution data assimilation

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Overview

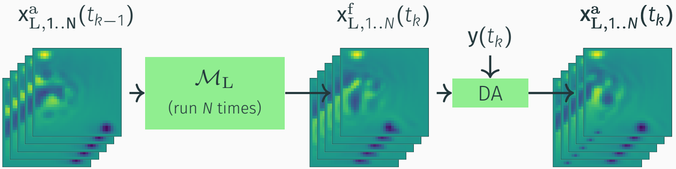
Objectives

The objectives are twofold:

1. Emulating a High-Resolution (HR) EnKF while running the forecast step with a Low-Resolution (LR) model
   $\Rightarrow$ reduction of the computational cost of the EnKF
2. Taking advantage of HR observations and reducing LR model bias

Motivation and method

EnKF - Low Resolution (EnKF-LR)



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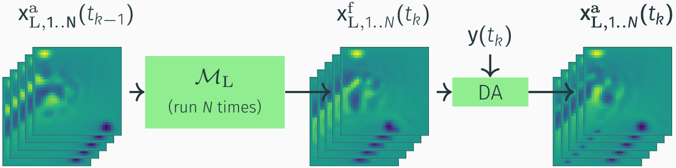

	EnKF-LR		
Observation error	High✘		
High-resolution processes	Poorly resolved✘		
Computational cost	Low✔		
Ensemble size	Large✔		
Error to the true $\mathbf{P}^{\mathrm{f}}$	Large✘		

Motivation and method

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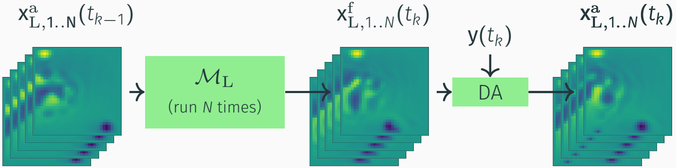
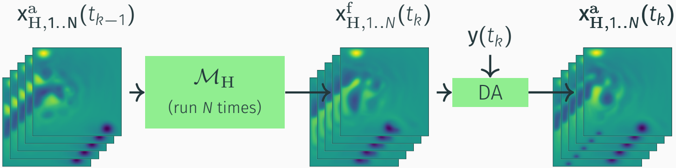
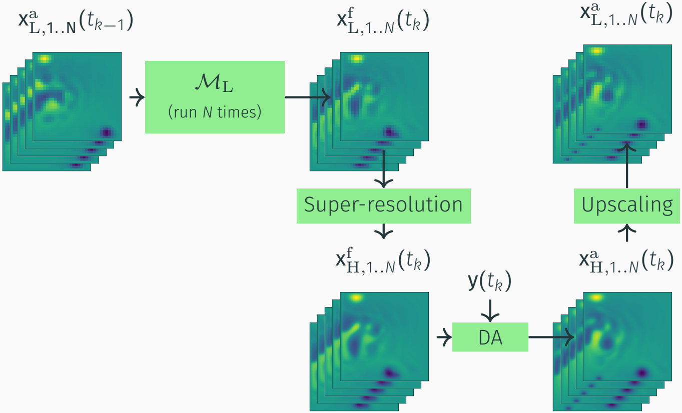


EnKF - High Resolution (EnKF-HR)



	EnKF-LR	EnKF-HR	
Observation error	High✔	Low✔	
High-resolution processes	Poorly resolved✔	Resolved✔	
Computational cost	Low✔	High, $\mathcal{O}\left(n^3\right)$✔	
Ensemble size	Large✔	Small✔	
Error to the true $\mathbf{P}^{\mathrm{f}}$	Large✔	Small✔	

EnKF - Super-resolution data assimilation (SRDA)



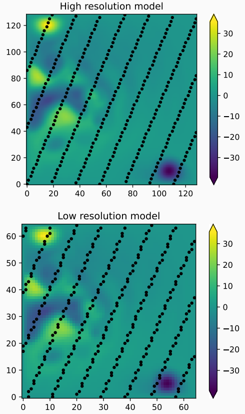
	EnKF-LR	EnKF-HR	SRDA
Observation error	High✔	Low✔	Low✔
High-resolution processes	Poorly resolved✔	Resolved✔	Emulated✔
Computational cost	Low✔	High, $\mathcal{O}(n^3)$✔	Low✔
Ensemble size	Large✔	Small✔	Large✔
Error to the true P^{f}	Large✔	Small✔	Medium✔

Model used

- Model used: Quasi-geostrophic model [Sakov & Oke, 2008]

Configuration	State size	Cost
HR	129×129	C
LR	65×65	$C/8$

- Observations:
  - True value perturbed by a Gaussian noise of standard deviation 2
  - Available every $\Delta t = 12$
  - Located along simulated satellite tracks (black dots on the figures)
  - Note the representativeness errors.



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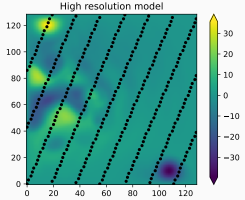
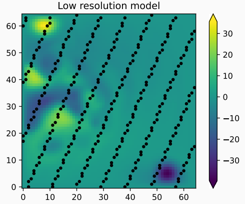


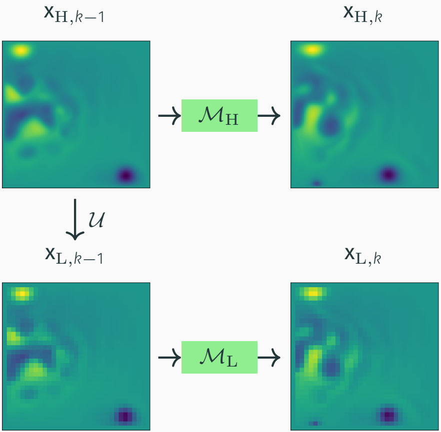


Super-resolution: downscaling operator

- A simple cubic spline interpolation (cubic)
- A neural network (NN) $\Rightarrow$ corrects the LR model error

Training set for the neural network

- Run one simulation of the HR model.
- Assemble matching pairs of LR and HR states: $(\mathrm{x}_{\mathrm{L},k}, \mathrm{x}_{\mathrm{H},k})$



$\mathcal{U}$: Upscaling (subsampling operator)

Training set for the neural network

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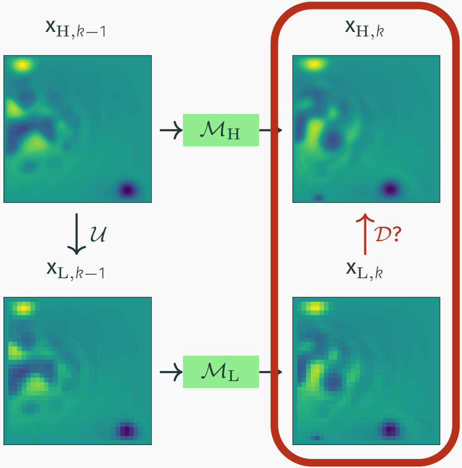


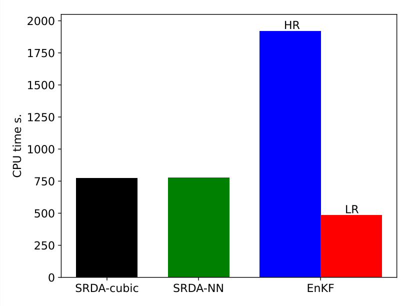
$\mathcal{U}$: Upscaling (subsampling operator)
$\mathcal{D}$: Downscaling (Neural network)

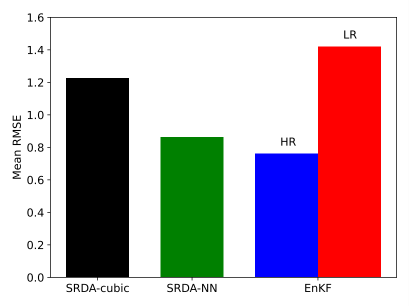
- Number of pairs: 10,000
- 8000 for training / 2000 for validation
- Architecture of the enhanced deep super-resolution network (EDSR) [Lim *et al.*, 2017]
- Training: minimization of the mean absolute error

Overview

DA experiments at fixed ensemble size

- Synthetic experiments with 500 assimilation cycles and 25 members





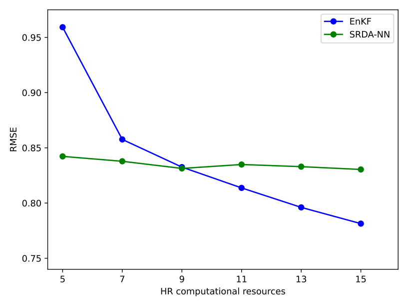
- SRDA-NN provides a good compromise between RMSE and computational cost.

DA experiments at a equivalent computational cost of integration

Trade-off: 1 HR member $\approx$ 8 LR members (integration time)

Design of the experiments:

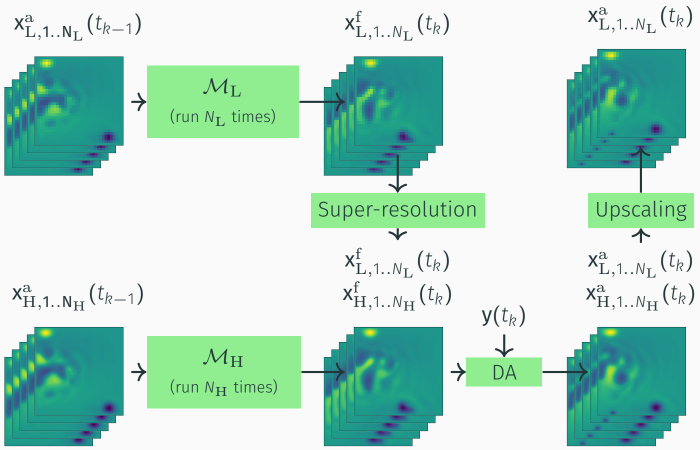
- Twin experiments with 500 assimilation cycles;
- Compared performance at equivalent computational cost of $\approx 5, 7, \ldots, 15$ HR members $\rightarrow$ with $40, 56, \ldots, 120$ LR members for the SRDA
- Localization and inflation tuned to optimal performance



- The SRDA improves the mean RMSE for limited computational resources
- For larger resources, the error from the emulator is the bottleneck
- Can we take the best of both worlds?

Overview

Hybrid covariance Super-Resolution Data Assimilation



	EnKF-LR	EnKF-HR	SRDA	Hybrid SRDA
Observation error	High✔	Low✔	Low✔	Low✔
HR processes	Poorly resolved✔	Resolved✔	Emulated✔	Emulated (LR)✔/ resolved (HR)✔
Computational cost	Low✔	High, $\mathcal{O}\left(n^3\right)$✔	Low✔	Customizable(✔–✔)
Ensemble size	Big✔	Small✔	Big✔	Big✔
Error to the true P^f	Large✔	Small✔	Medium✔	Customizable(✔–✔)

The covariance matrix $\mathsf{P}^{\mathrm{f}}_{\mathrm{h}}$ in the Hybrid SRDA is a linear combination of:

- $\mathsf{P}^{\mathrm{f}}_{\mathrm{HR}}$ computed from the HR ensemble;
- $\mathsf{P}^{\mathrm{f}}_{\mathrm{LR}}$ computed from the LR ensemble **downscaled** to the HR grid:

$$\mathsf{P}^{\mathrm{f}}_{\mathrm{h}} = (1-\alpha)\mathsf{P}^{\mathrm{f}}_{\mathrm{HR}} + \alpha\mathsf{P}^{\mathrm{f}}_{\mathrm{LR}}, \qquad 0 \leq \alpha \leq 1. \tag{1}$$

- $\alpha = 0$ full HR case $\rightarrow$ EnKF-HR
- $\alpha = 1$ full LR case $\rightarrow$ EnKF-LR
- Downscaling method “cubic spline interpolation” – **MRDA**

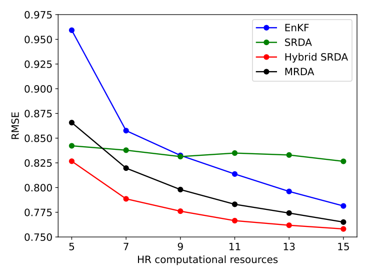
  $\Rightarrow$ Mixed-resolution ensemble data assimilation [Rainwater & Hunt, 2013].
- Downscaling method NN – **Hybrid SRDA (NN)**
- Results computed over the HR ensemble unless otherwise stated.

Fixed integration cost: trade-off HR/LR ensemble sizes experiments

Trade off: 1 HR member $\approx$ 8 LR members (integration time)

Design of the experiments: Twin experiments (500 assim. cycles) with parameters and hybrid coefficients optimally tuned depending on the computational resources:

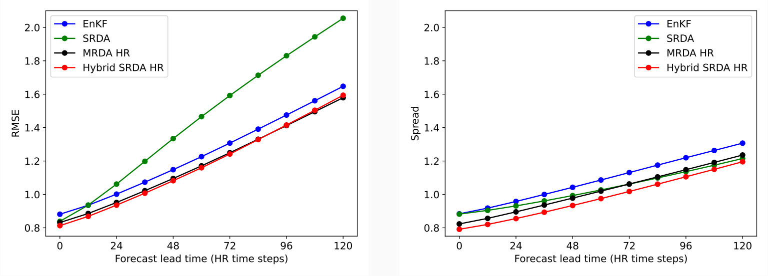
- **EnKF-HR** with $N_{\mathrm{H}} = 10$ members, **SRDA-NN** with $N_{\mathrm{L}} = 80$ members
- At equivalent computational cost, **Hybrid SRDA-NN** with $(N_{\mathrm{H}}, N_{\mathrm{L}}) = (2, 64), (3, 56), \ldots, (9, 8)$
- We repeat the comparison for HR resources ranging from 5 to 15 members



Design of the experiments:

- Twin experiments (500 assim. cycles) with optimal parameters for 7 HR computational resources
- At each assimilation cycle a 120 HR time steps (10 DA cycles) forecast is performed

Method	EnKF	Hybrid SRDA	MRDA	SRDA
Ensemble size	7 HR	4 HR	5 HR	56 LR



(a) (b)

- ▶ Divergence of the RMSE of the SRDA because of the low-resolution
- ▶ The RMSE (Fig. a) grows faster than the spread (Fig .b) because of the model error.

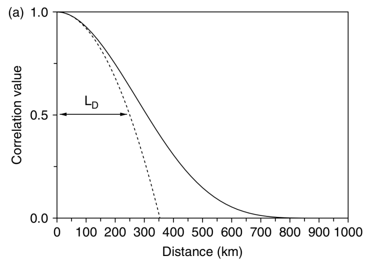
Characterization of the influence of each scheme on the covariance matrix

The background error covariance matrix decomposes as:

$$\mathrm{P}^{\mathrm{f}} = \Sigma \mathrm{C}^{\mathrm{f}} \Sigma, \tag{2}$$

where:

- Σ is a diagonal matrix with standard-deviation on the diagonal.
- C^{f} is the background error correlation matrix.



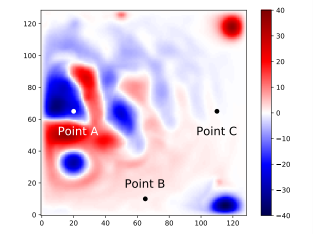
The spatial extent of the correlation functions can estimated through the **correlation length scale** L_p, [Pannekoucke *et al.*, 2007]:

$$L_p = \frac{\delta x}{\sqrt{-2 \ln \left(\rho \left(\delta x \right) \right)}} \tag{3}$$

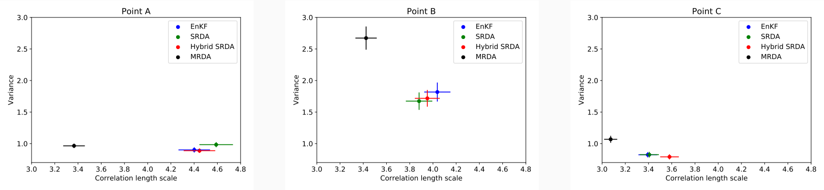
where ρ is a correlation function (column of C^{f})

Source: [Pannekoucke *et al.*, 2007]

Variance and correlation length scale

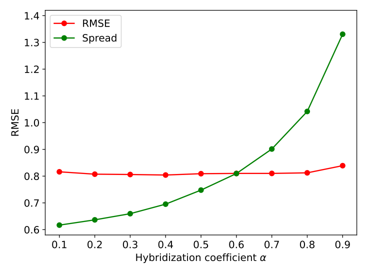


- **Point A:** high spatio-temporal variability
- **Point B:** passing of eddies
- **Point C:** low spatio-temporal variability

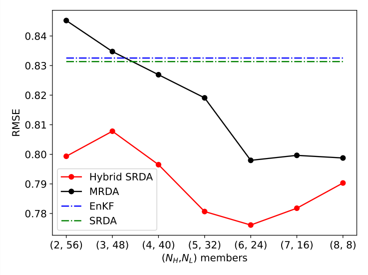


- **Underestimation** of the correlation length scale by the MRDA at all points
- **Overestimation** of the variance by the MRDA at point B
- The neural network allows for preserving the characteristics of the covariance matrix

Sensitivity to the hybridization coefficient α and the ensembles' size N_{H} & N_{L}



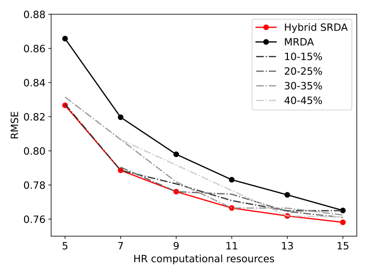
(a)



(b)

- **Figure a**
  - The RMSE is **stable** with respect to α.
  - The spread is very **sensitive** to α.
  - The method is tuned for the spread and RMSE of the HR ensemble to match.
- **Figure b**
  - Hybrid SRDA more stable than MRDA for different $(N_{\mathrm{H}}, N_{\mathrm{L}})$

Tuning of the method



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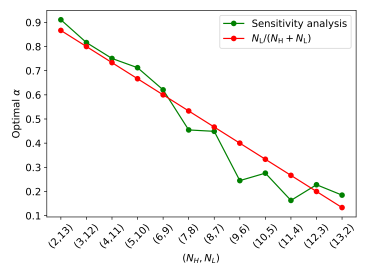

(b)

- **Figure a:** The Hybrid SRDA systematically outperforms the MRDA when the ratio $N_{\mathrm{H}} / (N_{\mathrm{H}} + N_{\mathrm{L}})$ is between 10 and 45%.
- **Figure b:** "Intuitive" method for the tuning of α

▸ **Possible other methods:**

- Optimal filtering of sample covariance [Ménétrier *et al.*, 2015]
- Spatio-temporal varying adaptive algorithm for α [Gharamti, 2021]

Overview

Conclusions/perspectives

Main result

▶ The **Hybrid SRDA** outperforms the SRDA, the EnKF and the MRDA at an equivalent computational cost.

▶ The method is highly customizable and makes optimal use of the computational resources available at hands.

▶ The effort to implement (Hybrid) SRDA in an existing DA system is minimal:

- No need to modify existing models
- Minor modifications to the DA code in case of Hybrid SRDA

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- The **Hybrid SRDA** outperforms the SRDA, the EnKF and the MRDA at an equivalent computational cost.
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- The effort to implement (Hybrid) SRDA in an existing DA system is minimal:
  - No need to modify existing models
  - Minor modifications to the DA code in case of Hybrid SRDA

Perspectives

- Application of SRDA in TOPAZ system $\longrightarrow$ work Antoine Bernigaud (NERSC) supervised by Julien Brajard & Laurent Bertino
- Application to the Norwegian Climate Prediction Model (NorCPM) within the project EU-Impetus4Change
- Do not throw away your old coarse-resolution models!

SRDA & Hybrid SRDA papers available on *Ocean Dynamics*!

SRDA

Hybrid SRDA

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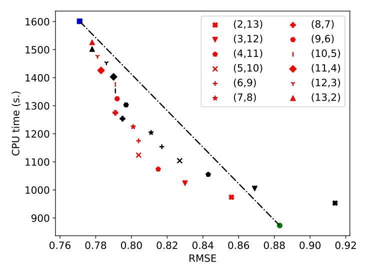
Fixed assimilation cost: fixed ensemble size & computational efficiency

- Twin experiments with 500 assimilation cycles
- Experiments at fixed ensemble size: (N_H, N_L) such that $N_H + N_L = 15$
- Optimal inflation and localization

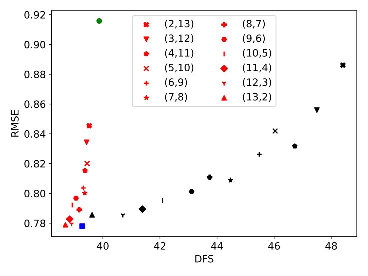


Hybrid SRDA-cubic	Hybrid SRDA-NN
EnKF	SRDA-NN

- The method is cost effective if it is under the black dashed line;

Performance in terms of degrees of freedom of the signal

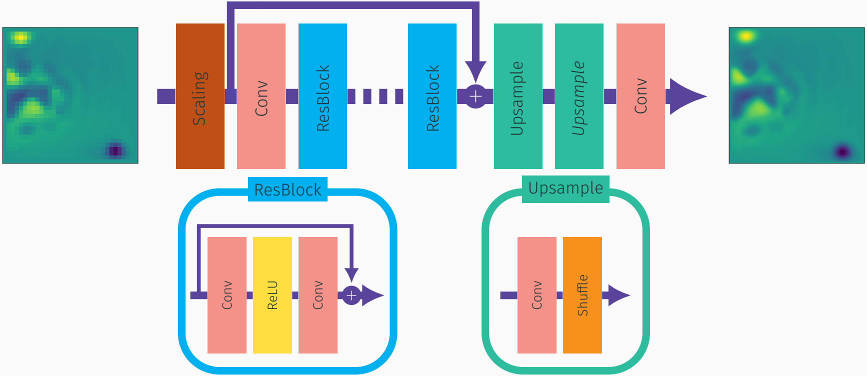
- **We seek a system that achieves the lowest error, doing minimal corrections.**
- DFS $= \mathrm{Tr}\,(\mathbf{KH}) \Rightarrow$ quantify the number of degrees of freedom reduced from the ensemble. [Cardinali *et al.*, 2004].
- Example of DFS with the different method at equivalent 15 members



MRDA	Hybrid SRDA-NN
EnKF	SRDA-NN

- The Hybrid SRDA-NN yields lower DFS (assimilation updates) than the MRDA.

Setup of the neural network



Architecture of the enhanced deep super-resolution network (EDSR) [?]

Training set for the neural network

- Run one simulation of the HR model.
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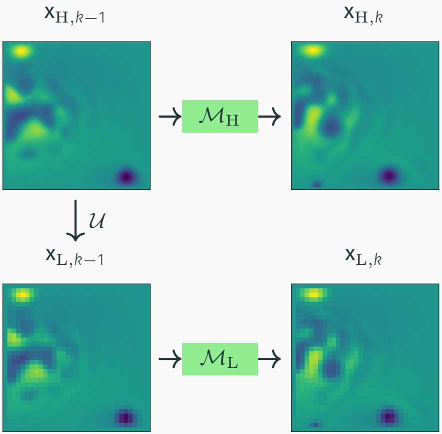

$\mathcal{U}$: Upscaling (subsampling operator)

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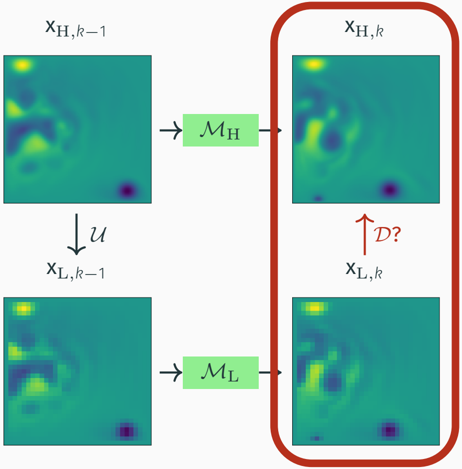


$\mathcal{U}$: Upscaling (subsampling operator)
$\mathcal{D}$: Downscaling (Neural network)

- Number of pairs: 10,000
- 8000 for training / 2000 for validation
- Architecture of the enhanced deep super-resolution network (EDSR) [?]
- Training: minimization of the mean absolute error

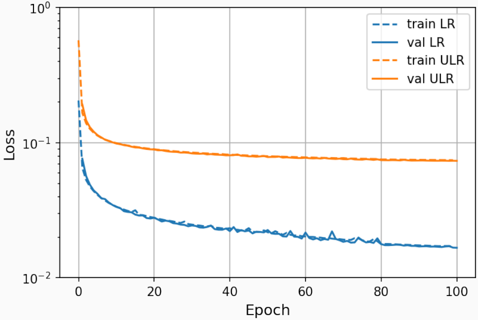
Training of the neural network

Minimize the mean absolute error (MAE):

$$L(\mathbf{w}) = \sum_{k=1}^{K} \sum_{i=1}^{S} \left| \mathcal{D}(\mathbf{x}_{\mathrm{L},k})_i - x_{\mathrm{H},k,i} \right| ,$$

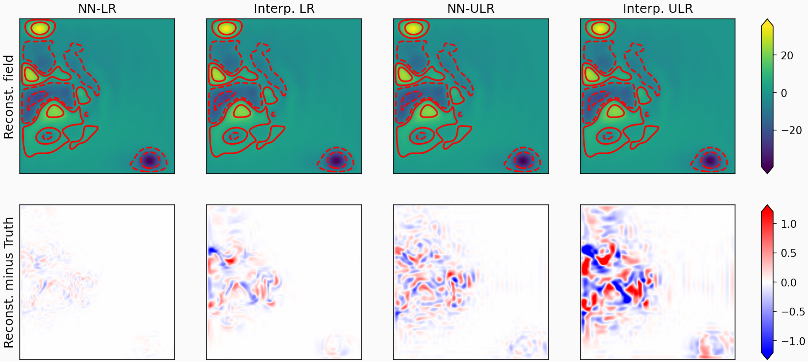
i:	the pixel index
S:	size of the state (129×129)
K:	size of the training set (K=8000)
$\mathbf{w}$:	weights of the neural network ($\sim 20,000$)

Training curve



Downscaling performance

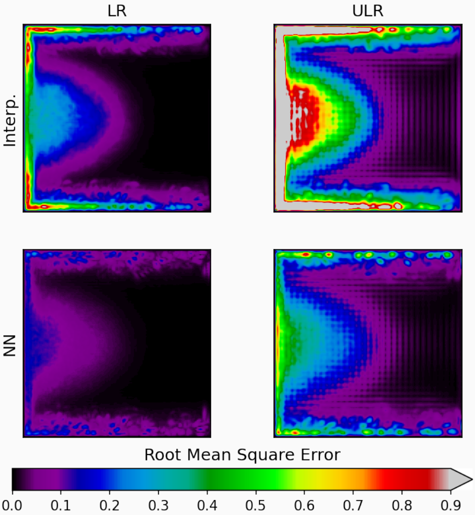
▶ Illustration with one typical sample



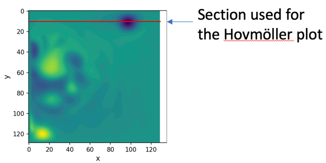
red lines: Contour of the true HR state

Downscaling performance (2)

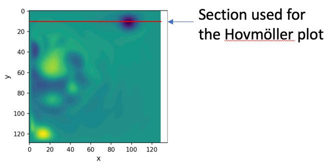
- Score on the validation dataset

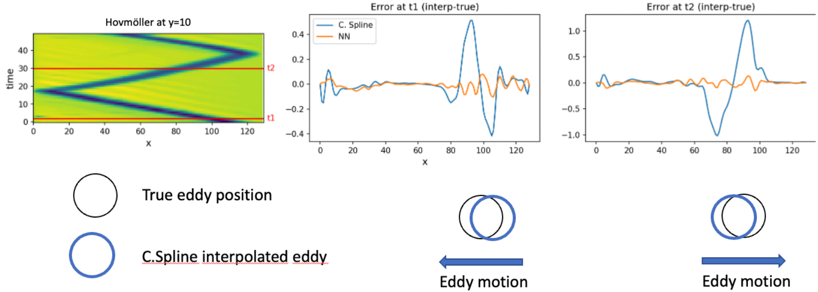


Model error correction



Model error correction

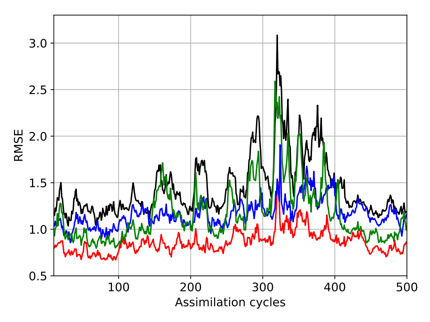




- Eddy propagation slower in the LR model
- The NN is smart enough to learn that

Reformulating the SRDA as a LR scheme

- We can reformulate the SRDA into LR EnKF equations so that we can separate the contributions from:
  1. the model error correction;
  2. the super-resolution observation operator (representativeness).



- Model error correction improves performance during challenging events
- Super-resolution obs. operator reduces error over the whole period

EnKF-LR
SRDA only with model error correction
Complete SRDA-NN
SRDA only with the super-res. observation operator